

**Module 5:
Population Forecasting**

**Lecture 5:
Population Forecasting**

NPTEL IIT Kharagpur

5. POPULATION FORECASTING

Design of water supply and sanitation scheme is based on the projected population of a particular city, estimated for the design period. Any underestimated value will make system inadequate for the purpose intended; similarly overestimated value will make it costly. Change in the population of the city over the years occurs, and the system should be designed taking into account of the population at the end of the design period.

Factors affecting changes in population are:

- increase due to births
- decrease due to deaths
- increase/ decrease due to migration
- increase due to annexation.

The present and past population record for the city can be obtained from the census population records. After collecting these population figures, the population at the end of design period is predicted using various methods as suitable for that city considering the growth pattern followed by the city.

5.1 ARITHMETICAL INCREASE METHOD

This method is suitable for large and old city with considerable development. If it is used for small, average or comparatively new cities, it will give low result than actual value. In this method the average increase in population per decade is calculated from the past census reports. This increase is added to the present population to find out the population of the next decade. Thus, it is assumed that the population is increasing at constant rate.

Hence, $dP/dt = C$ i.e. rate of change of population with respect to time is constant.

Therefore, Population after n^{th} decade will be $P_n = P + n.C$

Where, P_n is the population after n decade and P is present population.

Example:1

Predict the population for the year 2021, 2031, and 2041 from the following population data.

Year	1961	1971	1981	1991	2001	2011
Population	8,58,545	10,15,672	12,01,553	16,91,538,	20,77,820,	25,85,862

Solution

Year	Population	Increment
1961	858545	-
1971	1015672	157127
1981	1201553	185881
1991	1691538	489985
2001	2077820	386282
2011	2585862	508042

Average increment = 345463

Population in year 2021 is, $P_{2021} = 2585862 + 345463 \times 1 = 2931325$

Similarly, $P_{2031} = 2585862 + 345463 \times 2 = 3276788$

$P_{2041} = 2585862 + 345463 \times 3 = 3622251$

5.2 GEOMETRICAL INCREASE METHOD (OR GEOMETRICAL PROGRESSION METHOD)

In this method the percentage increase in population from decade to decade is assumed to remain constant. Geometric mean increase is used to find out the future increment in population. Since this method gives higher values and hence should be applied for a new industrial town at the beginning of development for only few decades. The population at the end of n^{th} decade 'P_n' can be estimated as:

$$P_n = P (1 + I_G/100)^n$$

Where, I_G = geometric mean (%)

P = Present population

N = no. of decades.

Example : 2

Considering data given in example 1 predict the population for the year 2021, 2031, and 2041 using geometrical progression method.

Solution

Year	Population	Increment	Geometrical increase Rate of growth
1961	858545	-	
1971	1015672	157127	$(157127/858545)$ = 0.18
1981	1201553	185881	$(185881/1015672)$ = 0.18
1991	1691538	489985	$(489985/1201553)$ = 0.40
2001	2077820	386282	$(386282/1691538)$ = 0.23
2011	2585862	508042	$(508042/2077820)$ = 0.24

$$\text{Geometric mean } I_G = (0.18 \times 0.18 \times 0.40 \times 0.23 \times 0.24)^{1/4}$$

$$= 0.235 \text{ i.e., } 23.5\%$$

$$\text{Population in year 2021 is, } P_{2021} = 2585862 \times (1 + 0.235)^1 = 3193540$$

Similarly for year 2031 and 2041 can be calculated by,

$$P_{2031} = 2585862 \times (1 + 0.235)^2 = 3944021$$

$$P_{2041} = 2585862 \times (1 + 0.235)^3 = 4870866$$

5.3 INCREMENTAL INCREASE METHOD

This method is modification of arithmetical increase method and it is suitable for an average size town under normal condition where the growth rate is found to be in increasing order. While adopting this method the increase in increment is considered for calculating future population. The incremental increase is determined for each decade from the past population and the average value is added to the present population along with the average rate of increase.

$$\text{Hence, population after } n^{\text{th}} \text{ decade is } P_n = P + n.X + \{n(n+1)/2\}.Y$$

Where, P_n = Population after n^{th} decade

X = Average increase

Y = Incremental increase

Example : 3

Considering data given in example 1 predict the population for the year 2021, 2031, and 2041 using incremental increase method.

Year	Population	Increase (X)	Incremental increase (Y)
1961	858545	-	-
1971	1015672	157127	-
1981	1201553	185881	+28754
1991	1691538	489985	+304104
2001	2077820	386282	-103703
2011	2585862	508042	+121760
	Total	1727317	350915
	Average	345463	87729

$$\begin{aligned} \text{Population in year 2021 is, } P_{2021} &= 2585862 + (345463 \times 1) + \left\{ \frac{1(1+1)}{2} \right\} \times 87729 \\ &= 3019054 \end{aligned}$$

$$\begin{aligned} \text{For year 2031 } P_{2031} &= 2585862 + (345463 \times 2) + \left\{ \frac{2(2+1)}{2} \right\} \times 87729 \\ &= 3539975 \end{aligned}$$

$$\begin{aligned} P_{2041} &= 2585862 + (345463 \times 3) + \left\{ \frac{3(3+1)}{2} \right\} \times 87729 \\ &= 4148625 \end{aligned}$$

5.4 GRAPHICAL METHOD

In this method, the populations of last few decades are correctly plotted to a suitable scale on graph. The population curve is smoothly extended for getting future population. This extension should be done carefully and it requires proper experience and judgment. The best way of applying this method is to extend the curve by comparing with population curve of some other similar cities having the similar growth condition.

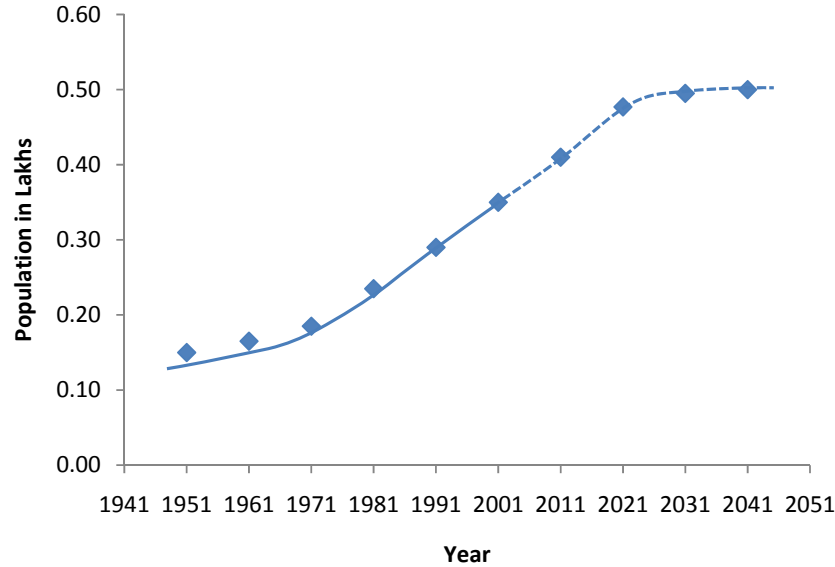


Figure 5.1 Graphical method of population forecasting

5.5 COMPARATIVE GRAPHICAL METHOD

In this method the census populations of cities already developed under similar conditions are plotted. The curve of past population of the city under consideration is plotted on the same graph. The curve is extended carefully by comparing with the population curve of some similar cities having the similar condition of growth. The advantage of this method is that the future population can be predicted from the present population even in the absence of some of the past census report. The use of this method is explained by a suitable example given below.

Example: 4

Let the population of a new city X be given for decades 1970, 1980, 1990 and 2000 were 32,000; 38,000; 43,000 and 50,000, respectively. The cities A, B, C and D were developed in similar conditions as that of city X. It is required to estimate the population of the city X in the years 2010 and 2020. The population of cities A, B, C and D of different decades were given below:

- (i) City A was 50,000; 62,000; 72,000 and 87,000 in 1960, 1972, 1980 and 1990, respectively.

- (ii) City B was 50,000; 58,000; 69,000 and 76,000 in 1962, 1970, 1981 and 1988, respectively.
- (iii) City C was 50,000; 56,500; 64,000 and 70,000 in 1964, 1970, 1980 and 1988, respectively.
- (iv) City D was 50,000; 54,000; 58,000 and 62,000 in 1961, 1973, 1982 and 1989, respectively.

Population curves for the cities A, B, C, D and X were plotted. Then an average mean curve is also plotted by dotted line as shown in the figure. The population curve X is extended beyond 50,000 matching with the dotted mean curve. From the curve the populations obtained for city X are 58,000 and 68,000 in year 2010 and 2020.

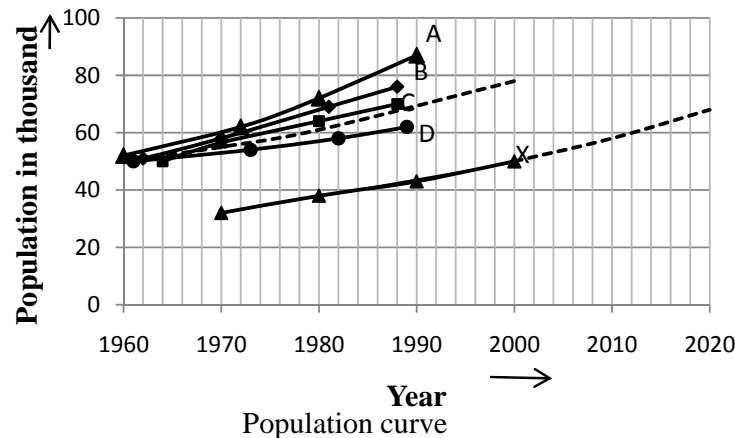


Figure 5.2 Comparative graph method

5.6 MASTER PLAN METHOD

The big and metropolitan cities are generally not developed in haphazard manner, but are planned and regulated by local bodies according to master plan. The master plan is prepared for next 25 to 30 years for the city. According to the master plan the city is divided into various zones such as residence, commerce and industry. The population densities are fixed for various zones in the master plan. From this population density total water demand and wastewater generation for that zone can be worked out. So by this method it is very easy to access precisely the design population.

5.7 LOGISTIC CURVE METHOD

This method is used when the growth rate of population due to births, deaths and migrations takes place under normal situation and it is not subjected to any extraordinary changes like epidemic, war, earth quake or any natural disaster etc. the population follow the growth curve characteristics of living things within limited space and economic opportunity. If the population of a city is plotted with respect to time, the curve so obtained under normal condition is look like S-shaped curve and is known as logistic curve.

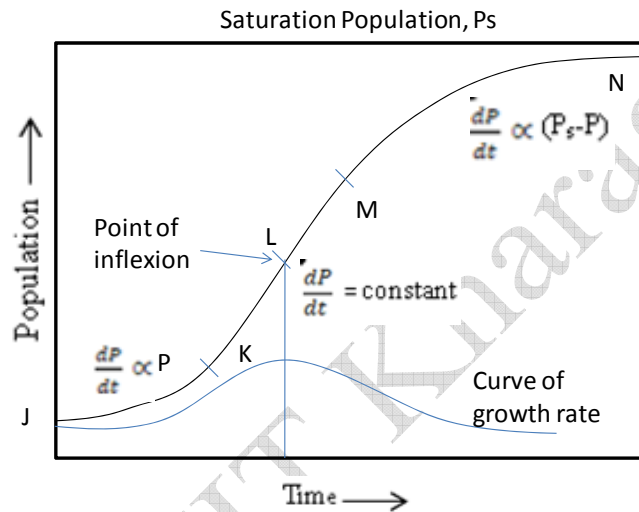


Figure 5.3 Logistic curve for population growth

In figure, the curve shows an early growth JK at an increasing rate i.e. geometric growth or log growth, $\frac{dP}{dt} \propto P$, the transitional middle curve KM follows arithmetic increase i.e. $\frac{dP}{dt} = \text{constant}$ and later growth MN the rate of change of population is proportional to difference between saturation population and existing population, i.e. $\frac{dP}{dt} \propto (P_s - P)$. Verhulst has put forward a mathematical solution for this logistic curve JN which can be represented by an autocatalytic first order equation, given by

$$\log_e \left(\frac{P_s - P}{P} \right) - \log_e \left(\frac{P_s - P_0}{P_0} \right) = -K \cdot P_s \cdot t$$

where, P = Population at any time t from the origin J

P_s = Saturation population

P_0 = Population of the city at the start point J

K = Constant

t = Years

From the above equation we get

$$\log_e \left(\frac{P_s - P}{P} \right) \left(\frac{P_0}{P_s - P_0} \right) = -K.P_s.t$$

After solving we get,

$$P = \frac{P_s}{1 + \frac{P_s - P_0}{P_0} \log_e^{-1}(-K.P_s.t)}$$

Substituting $\frac{P_s - P_0}{P_0} = m$ (a constant)

and $-K.P_s = n$ (another constant)

we get $P = \frac{P_s}{1 + m \log_e^{-1}(n.t)}$

This is the required equation of the logistic curve, which will be used for predicting population. McLean further suggested that if only three pairs of characteristic values P_0, P_1, P_2 at times $t = t_0 = 0, t_1$ and $t_2 = 2t_1$ extending over the past record are chosen, the saturation population P_s and constant m and n can be estimated by the following equation, as follows:

$$P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2}$$

$$m = \frac{P_s - P_0}{P_0}$$

$$n = \frac{2.3}{t_1} \log_{10} \left(\frac{P_0(P_s - P_1)}{P_1(P_s - P_0)} \right)$$

Example: 5

The population of a city in three consecutive years i.e. 1991, 2001 and 2011 is 80,000; 250,000 and 480,000, respectively. Determine (a) The saturation population, (b) The equation of logistic curve, (c) The expected population in 2021.

Solution

It is given that

$$P_0 = 80,000 \quad t_0 = 0$$

$$P_1 = 250,000 \quad t_1 = 10 \text{ years}$$

$$P_2 = 480,000 \quad t_2 = 20 \text{ years}$$

The saturation population can be calculated by using equation

$$P_s = \frac{2P_0P_1P_2 - P_1^2(P_0 + P_2)}{P_0P_2 - P_1^2}$$

$$= \frac{2 \times 80,000 \times 2,50,000 \times 4,80,000 - 2,50,000 \times 2,50,000 \times (80,000 + 4,80,000)}{80,000 \times 4,80,000 - 2,50,000 \times 2,50,000}$$

$$= 655,602$$

$$\text{We have } m = \frac{P_s - P_0}{P_0} = \frac{655,602 - 80,000}{80,000} = 7.195$$

$$n = \frac{2.3}{t_1} \log_{10} \frac{P_0(P_s - P_1)}{P_1(P_s - P_0)}$$

$$= \frac{2.3}{10} \log_{10} \left(\frac{80,000(655,602 - 2,50,000)}{250,000(655,602 - 80,000)} \right)$$

$$= -0.1488$$

Population in 2021

$$P = \frac{P_s}{1 + m \log_e^{-1}(n.t)}$$

$$= \frac{6,55,602}{1 + 7.195 \times \log_e^{-1}(-0.1488 \times 30)}$$

$$= \frac{6,55,602}{1 + 7.195 \times 0.0117} = 605,436$$

Questions

1. Explain different methods of population forecasting.
2. The population data for a town is given below. Find out the population in the year 2021, 2031 and 2041 by (a) arithmetical (b) geometric (c) incremental increase methods.

Year	1971	1981	1991	2001	2011
Population	84,000	1, 15,000	1, 60,000	2, 05,000	2, 50,000

3. In three consecutive decades the population of a town is 40,000; 100,000 and 130,000. Determine: (a) Saturation population; (b) Equation for logistic curve; (c) Expected population in next decade.

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